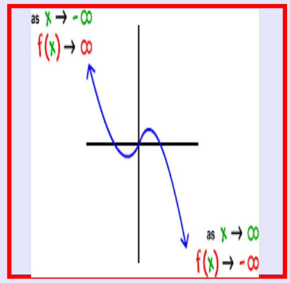
Math 245

Spring 2022

Lecture 42



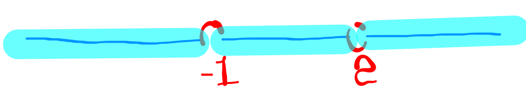
Given $f(x) = \dfrac{x^2 - 4}{x^2 - x - 2}$

1) Y-Int $\Rightarrow x = 0 \Rightarrow f(0) = 2 \Rightarrow (0, 2)$

2) x-Int $\Rightarrow y = 0 \Rightarrow f(x) = 0 \Rightarrow x - 4 = 0 \Rightarrow x = 4 \Rightarrow (4, 0)$

3) Domain $\Rightarrow x^2 - x - 2 \neq 0 \Rightarrow (x-2)(x+1) \neq 0$

$x \neq 2, \quad x \neq -1$



$(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

4) V.A. $x = 2, \ x = -1$

5) H.A. $y = 0$

Deg. of num. < Deg. of Deno

If $x=5$

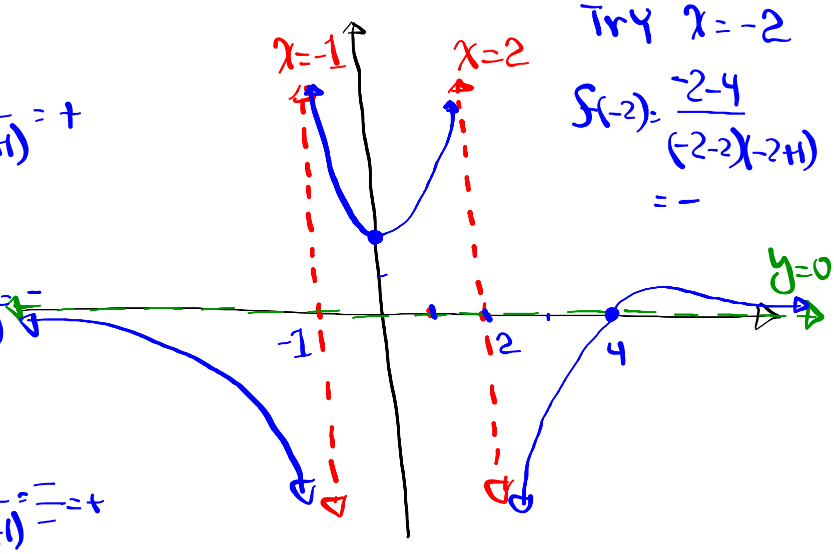
$f(5)=\dfrac{5-4}{(5-2)(5+1)}=+$

If $x=3$

$f(3)=\dfrac{3-4}{(3-2)(3+1)}=-$

If $x=1$

$f(1)=\dfrac{1-4}{(1-2)(1+1)}=\dfrac{-}{-}=+$



Try $x=-2$

$f(-2)=\dfrac{-2-4}{(-2-2)(-2+1)}$

$=-$

$f(x)=\dfrac{x^2}{x^2+1}$

Y-Int $\rightarrow f(0)=0 \Rightarrow (0,0)$

x-Int $\rightarrow f(x)=0 \Rightarrow x^2=0 \rightarrow x=0 \Rightarrow (0,0)$

Domain: All Reals since $x^2+1 \neq 0$

V.A. None

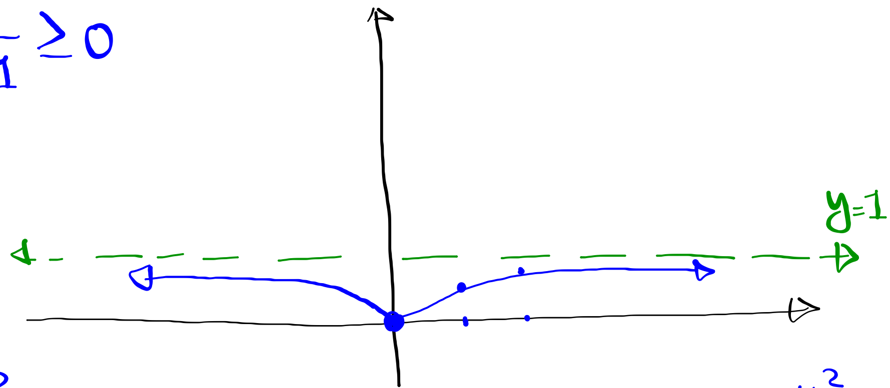
H.A. $y=\dfrac{1}{1}$ Since Deg. of num. = Deg. of Deno.

$y=1$

$f(-x)=\dfrac{(-x)^2}{(-x)^2+1}=\dfrac{x^2}{x^2+1}=f(x) \Rightarrow$ even function

Symmetric with respect to Y-axis.

$f(x) = \frac{x^2}{x^2+1} \geq 0$



$f(1) = \frac{1^2}{1^2+1} = \frac{1}{2}$

$f(2) = \frac{2^2}{2^2+1} = \frac{4}{5}$

$f(10) = \frac{10^2}{10^2+1} = \frac{100}{101}$

Range $[0, 1)$

find

1) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

2) $7! - 5! = \underbrace{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} - \underbrace{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= 5040 - 120 = \boxed{4920}$

3) $\frac{8!}{7!} = \frac{8 \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \boxed{8}$

4) $\frac{5!}{4! \cdot 1!} = \frac{5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1} \cdot 1} = \frac{5}{1} = \boxed{5}$

5) $_6C_2 = \frac{6!}{2!\cdot(6-2)!} = \frac{6!}{2!\cdot 4!} = \frac{\cancel{6}^{3}\cdot 5\cdot \cancel{4!}}{\cancel{2}\cdot 1\cdot \cancel{4!}} = \boxed{15}$

$_nC_r = \frac{n!}{r!(n-r)!}$

6) $\binom{6}{2} = \frac{6!}{2!\cdot(6-2)!} = \frac{6!}{2!\cdot 4!} = \frac{\cancel{6}^{3}\cdot 5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{2}\cdot 1\cdot \cancel{4\cdot 3\cdot 2\cdot 1}} = \frac{15}{1} = \boxed{15}$

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

7) $_8C_3 = \frac{8!}{3!\cdot(8-3)!} = \frac{8!}{3!\cdot 5!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{3\cdot 2}\cdot 1\cdot \cancel{5!}} = \frac{56}{1} = \boxed{56}$

8) $\binom{9}{4} = \frac{9!}{4!\cdot(9-4)!} = \frac{9!}{4!\cdot 5!} = \frac{9\cdot \cancel{8}^{2}\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{4}\cdot \cancel{3\cdot 2}\cdot 1\cdot \cancel{5!}} = \frac{9\cdot 2\cdot 7}{1} = \boxed{126}$

Binomial Expansion

Two-Terms

Assume $a+b \neq 0$

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

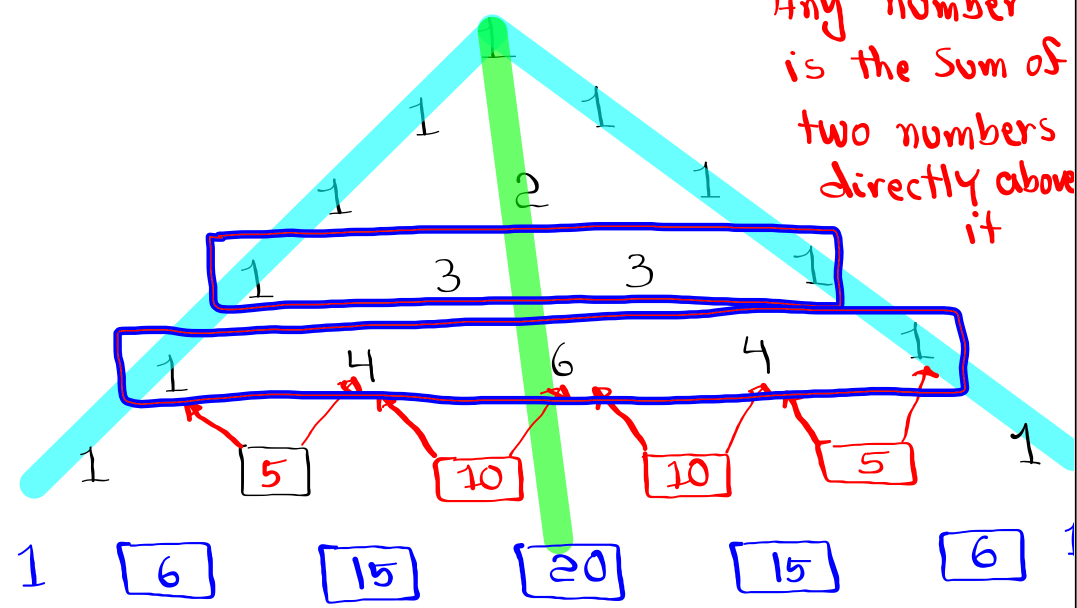
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\vdots \qquad\qquad \vdots$

May 3, 2022

Pascal Triangle

Any number is the Sum of two numbers directly above it



$(a+b)^n$

First term a^n, Last term b^n

There will be $(n+1)$ terms

From Left-to-right ⇒ Power of a decreases

" " b increases

$$a^n + a^{n-1}b^1 + a^{n-2}b^{\boxed{2}} + a^{n-3}b^{\boxed{3}} + \cdots + b^n$$

$$\binom{n}{0} \quad \binom{n}{1} \quad \binom{n}{2} \quad \binom{n}{3} \quad \cdots$$

So $\binom{n}{r} a^{n-r} b^r$ ← this is $(r+1)$th term.

$$(a+b)^3 =$$

$$\binom{3}{0}a^3 + \binom{3}{1}a^2b^1 + \binom{3}{2}a^1b^2 + \binom{3}{3}b^3$$

$$\boxed{1 \qquad 3 \qquad 3 \qquad 1}$$

$$\binom{3}{0} = \frac{3!}{0!\cdot(3-0)!} = \frac{3!}{0!\cdot 3!} = \frac{3!}{1\cdot 3!} = 1$$

$$\binom{3}{1} = \frac{3!}{1!\cdot(3-1)!} = \frac{3!}{1!\cdot 2!} = 3$$

$$\binom{3}{2} = \frac{3!}{2!\cdot(3-2)!} = \frac{3!}{2!\cdot 1!} = 3$$

$$\binom{3}{3} = \frac{3!}{3!\cdot(3-3)!} = \frac{3!}{3!\cdot 0!} = 1$$

$$(a+b)^4$$

$$\binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4$$

$$\boxed{1 \qquad 4 \qquad 6 \qquad 4 \qquad 1}$$

$$\binom{4}{2} = \frac{4!}{2!\cdot(4-2)!} = \frac{4!}{2!\cdot 2!} = \frac{\cancel{4}^{2}\cdot 3\cdot \cancel{2\cdot 1}}{\cancel{2}\cdot 1\cdot \cancel{2\cdot 1}} = 6$$